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Branching ratio and CP asymmetry of $B_s \to \pi^+\pi^-$ decays in the perturbative QCD approach

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Abstract

In this paper, we calculate the decay rate and CP asymmetry of the $B_s \to \pi^+\pi^-$ decay in perturbative QCD approach with Sudakov resummation. Since none of the quarks in final states is the same as those of the initial B_s meson, this decay can occur only via annihilation diagrams in the standard model. Besides the current-current operators, the contributions from the QCD and electroweak penguin operators are also taken into account. We find that (a) the branching ratio is about 4×10^{-7}; (b) the penguin diagrams dominate the total contribution; and (c) the direct CP asymmetry is small in size: no more than 3%; but the mixing-induced CP asymmetry can be as large as ten percent testable in the near future LHC-b experiments.

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I. INTRODUCTION

In recent years, rare B decays are attracting more and more attentions, since they provide a good opportunity for testing the Standard Model(SM), probing CP violation and searching for possible new physics beyond the SM. Since 1999, the data sample of the pair production and decays of B mesons collected by BaBar and Belle Collaborations is increased rapidly. In the future LHC-b experiments, B_s and B_C mesons can also be produced, and the rare B decays with a branching ratio around 10^{-7} can be observed. The rapid progress in current B factory experiments and the bright expectation in LHC-b experiments induced a great interest in the studies of rare decays of B meson.

The rare decay $B_s \to \pi^+\pi^-$ can occur only via annihilation diagrams in SM because none of quarks in final states is the same as those of the initial B_s meson. The usual method to treat non-leptonic decays of B meson is Factorization Approach(FA) [1], which has achieved great success in explaining many decay branching ratios [2, 3, 4]. However, this method failed in describing $B_s \to \pi^+\pi^-$ decay, because we need the $\pi \to \pi$ form factor at very large momentum transfer $\mathcal{O}(M_B)$. So far, little is known about the form factor at such a large momentum transfer in FA. In the QCD factorization approach [5], one cannot perform a real calculation of the annihilation diagrams, but estimating the annihilation amplitude by introducing a phenomenological parameter. In this paper, we calculate the branching ratio and CP asymmetries of $B_s \to \pi^+\pi^-$ decay by employing the perturbative QCD approach(PQCD) [6]. This method has been developed for the studies of the B meson decays [7] and successfully applied to calculate the annihilation diagrams [8, 9]. When the final states are light mesons such as pions, the perturbative QCD approach(PQCD) can be safely used because of asymptotic freedom of QCD [10].

In the next section, we give our theoretical formulas for the decay $B_s \to \pi^+\pi^-$ in PQCD framework. In section 3, we give the numerical results of the branching ratio of $B_s \to \pi^+\pi^-$ and discuss CP asymmetry of the decay.

II. PERTURBATIVE CALCULATIONS

The related effective Hamiltonian for the process $B_s \to \pi^+\pi^-$ is given by [9, 11]

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\{V_{ub}^* V_{us}\,[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] - V_{tb}^* V_{ts}\sum_{i=3}^{10} C_i(\mu)O_i(\mu)\}, \qquad (1)$$

where $C_i(\mu)(i=1,\cdots,10)$ are Wilson coefficients at the renormalization scale μ and the operators $O_i(i=1,\cdots,10)$ are

$$\begin{aligned}
O_1 &= (\bar{s}_i u_j)_{V-A}(\bar{u}_j b_i)_{V-A}, \\
O_2 &= (\bar{s}_i u_i)_{V-A}(\bar{u}_j b_j)_{V-A}, \\
O_3 &= (\bar{s}_i b_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V-A}, \\
O_4 &= (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V-A}, \\
O_5 &= (\bar{s}_i b_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V+A}, \\
O_6 &= (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V+A}, \\
O_7 &= \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V+A}, \\
O_8 &= \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A}, \\
O_9 &= \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V-A}, \\
O_{10} &= \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A}.
\end{aligned} \tag{2}$$

Here i and j are $SU(3)$ color indices, the sum over q runs over the quark field that are active at the scale $\mu = O(m_b)$, i.e., $q \in \{u, d, s, c, b\}$. Operators O_1, O_2 come from tree level, O_3, O_4, O_5, O_6 are QCD-Penguins operators and O_7, O_8, O_9, O_{10} come from electroweak-Penguins.

In the PQCD approach, the decay amplitude is separated into soft(Φ), hard(H), and harder(C) dynamics characterized by different scales. It is conceptually written as the following,

$$\text{Amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \ \mathrm{Tr}\big[C(t)\Phi_B(k_1)\Phi_\pi(k_2)\Phi_\pi(k_3)H(k_1,k_2,k_3,t)\big], \tag{3}$$

where k_i's are momenta of light quarks included in each mesons, and Tr denotes the trace over Dirac and color indices. $C(t)$ is Wilson coefficient which results from the radiative corrections at short distance. Φ_M is wave function which describes the hadronization of mesons. The wave functions should be universal and channel independent, we can use Φ_M which is determined by other ways. The hard part H is rather process-dependent. In the following, we start to compute the decay amplitude of $B_s \to \pi^+\pi^-$.

Since we set B_s at rest, in the light-cone coordinates, the momentum of the B_s, π^- and π^+ are written as :

$$P_1 = \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), P_2 = \frac{M_B}{\sqrt{2}}(1, 0, \mathbf{0}_T), P_3 = \frac{M_B}{\sqrt{2}}(0, 1, \mathbf{0}_T). \quad (4)$$

Denoting the light (anti-)quark momenta in B, π^- and π^+ as k_1, k_2 and k_3, respectively, we can choose:

$$k_1 = (x_1 p_1^+, 0, \mathbf{k}_{1T}), \quad k_2 = (x_2 p_2^+, 0, \mathbf{k}_{2T}), \quad k_3 = (0, x_3 p_3^-, \mathbf{k}_{3T}). \quad (5)$$

According to effective Hamiltonian(1), we draw the lowest order diagrams of $B_s \to \pi^+\pi^-$ in Fig.1. For the factorizable diagrams (a) and (b), we find their contributions cancel each other, which is a result of exact isospin symmetry. For the non-factorizable diagrams (c) and (d), the contribution comes from tree operator is

$$\begin{aligned}
M_a^T = & \frac{1}{\sqrt{2N_c}} 64\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1\, b_2 db_2\ \phi_B(x_1, b_1) \\
& \times \Big[C_2(t_1)\alpha_s(t_1) \big\{ -x_3 \phi_\pi^A(x_2)\phi_\pi^A(x_3) - r^2(x_2 + x_3)\phi_\pi^P(x_2)\phi_\pi^P(x_3) \\
& \quad + r^2(x_2 - x_3)\phi_\pi^P(x_2)\phi_\pi^T(x_3) + r^2(x_2 - x_3)\phi_\pi^P(x_3)\phi_\pi^T(x_2) \\
& \quad - r^2(x_2 + x_3)\phi_\pi^T(x_2)\phi_\pi^T(x_3) \big\} h_a^{(1)}(x_1, x_2, x_3, b_1, b_2) E_m(t_1) \\
& + C_2(t_2)\alpha_s(t_2) \big\{ x_2 \phi_\pi^A(x_2)\phi_\pi^A(x_3) + r^2(2 + x_2 + x_3)\phi_\pi^P(x_2)\phi_\pi^P(x_3) \\
& \quad + r^2(x_2 - x_3)\phi_\pi^P(x_2)\phi_\pi^T(x_3) + r^2(x_2 - x_3)\phi_\pi^T(x_2)\phi_\pi^P(x_3) \\
& \qquad + r^2(-2 + x_2 + x_3)\phi_\pi^T(x_2)\phi_\pi^T(x_3) \big\} h_a^{(2)}(x_1, x_2, x_3, b_1, b_2) E_m(t_2) \Big], \quad (6)
\end{aligned}$$

where $r = m_{0\pi}/m_B = m_\pi^2/[m_B(m_u + m_d)]$. $C_F = 4/3$ is the group factor of the $SU(3)_c$ gauge group. The expressions of the meson distribution amplitudes ϕ_M, the Sudakov factor $E_m(t_i)$ and the functions $h_a^{(1,2)}(x_1, x_2, x_3, b_1, b_2)$ are given in the appendix. The contribution of penguin-diagrams M_a^P can be obtained by replacing $C_2(t_i)(i = 1, 2)$ with

$$a^P(t_i) = 2C_4(t_i) + 2C_6(t_i) + \frac{1}{2}C_8(t_i) + \frac{1}{2}C_{10}(t_i), \quad (7)$$

in Eq.(6). The explicit expressions of QCD corrected Wilson coefficients C_2, C_4, C_6, C_8 and C_{10} as a function of scale t can be found in the Appendix of Ref.[9].

Now, the total decay amplitude for $B_s \to \pi^+\pi^-$ is given by

$$A = V_{ub}^* V_{us} M_a^T - V_{tb}^* V_{ts} M_a^P = V_{ub}^* V_{us} M_a^T [1 + z e^{i(\delta - \gamma)}], \quad (8)$$

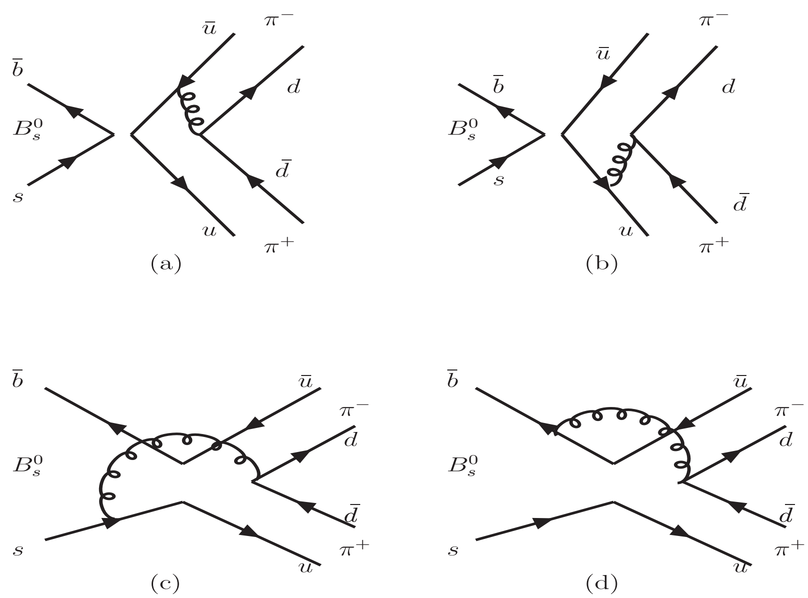

FIG. 1: The lowest order diagrams for $B_s^0 \to \pi^+\pi^-$ decay.

where $z = |V_{tb}^* V_{ts}/V_{ub}^* V_{us}||M_a^P/M_a^T|$, $V_{ub} \simeq |V_{ub}|e^{-i\gamma}$ and δ is the relative strong phase between tree diagrams(M_a^T) and penguin diagrams(M_a^P). z and δ can be calculated from PQCD.

The decay width is expressed as

$$\Gamma(B_s^0 \to \pi^+\pi^-) = \frac{G_F^2 M_B^3}{128\pi}|A|^2 = \frac{G_F^2 M_B^3}{128\pi}|V_{ub}^* V_{us} M_a^T|^2[1 + z^2 + 2z\cos(\delta - \gamma)]. \tag{9}$$

Similarly, we can get the decay width for $\bar{B}_s^0 \to \pi^+\pi^-$

$$\Gamma(\bar{B}_s^0 \to \pi^+\pi^-) = \frac{G_F^2 M_B^3}{128\pi}|\bar{A}|^2, \tag{10}$$

where

$$\bar{A} = V_{ub} V_{us}^* M_a^T - V_{tb} V_{ts}^* M_a^P = V_{ub} V_{us}^* M_a^T[1 + ze^{i(\delta+\gamma)}]. \tag{11}$$

III. NUMERICAL EVALUATION AND SUMMARY

The following parameters have been used in our numerical calculation:

$$M_{B_s} = 5.37 \text{ GeV}, \; m_{0\pi} = 1.4 \text{ GeV}, \; \Lambda_{QCD}^{f=4} = 0.25 \text{ GeV}, f_{B_s} = 236 \text{ MeV},$$
$$f_\pi = 130 \text{ MeV}, \; \tau_{B_s^0} = 1.46 \times 10^{-12}\text{s}, |V_{tb}^* V_{ts}| = 0.0395, |V_{ub}^* V_{us}| = 0.0008. \tag{12}$$

We leave the CKM phase angle γ as a free parameter to explore the branching ratio and CP asymmetry parameter dependence on it. In SM, the CKM phase angle is the origin of CP violation.

From Eq.(9) and (10), we get the averaged decay width for $B_s^0(\bar{B}_s^0) \to \pi^+\pi^-$

$$\begin{aligned}\Gamma(B_s^0(\bar{B}_s^0) \to \pi^+\pi^-) &= \frac{G_F^2 M_B^3}{128\pi}(|A|^2/2 + |\bar{A}|^2/2) \\ &= \frac{G_F^2 M_B^3}{128\pi}|V_{ub}^* V_{us} M_a^T|^2[1 + 2z\cos\gamma\cos\delta + z^2]. \end{aligned} \tag{13}$$

Using the above parameters, we get $z = 13.4$ and $\delta = 168°$ in PQCD. Eq.(13) is a function of CKM angle γ. In Fig. 2, we plot the averaged branching ratio of the decay $B_s^0(\bar{B}_s^0) \to \pi^+\pi^-$ with respect to the parameter γ. From Fig. 2, we can see that [18]:

$$\mathbf{Br}(B_s^0(\bar{B}_s^0) \to \pi^+\pi^-) = (4.2 \pm 0.6) \times 10^{-7}, \tag{14}$$

for $0 < \gamma < \pi$. The number $z = |V_{tb}^* V_{ts}/V_{ub}^* V_{us}||M_a^P/M_a^T| = 13.4$ means the amplitude of penguin diagrams is about 13.4 times more than that of tree diagrams. Therefore almost all the contribution comes from penguin diagrams in this decay and the branching ratio is not sensitive to γ.

In Ref.[17], Beneke et al have estimated the branching ratio for $B_s \to \pi^+\pi^-$ in QCD Factorization approach. In order to avoid the endpoint singularities, they introduced parameters to replace the divergent integral. In this approach, they estimated that the branching ratio of this decay is $(0.24 - 1.55) \times 10^{-7}$ with those phenomenological parameters. In our work, the calculation has no endpoint singularity because of k_T[6]. Our predicted result is larger than their simple estimation, which can be tested by the experiments.

For the experimental side, we notice that there is only upper limit of the decay $B_s^0 \to \pi^+\pi^-$ given at 90% confidence level [12]

$$\mathrm{Br}(B_s^0 \to \pi^+\pi^-) < 1.4 \times 10^{-4}. \tag{15}$$

Obviously, our predicted result is still far from this upper limit.

In SM, CP violation comes from interference between amplitudes with different CP eigenvalues. The strong interaction eigenstates B_s^0 and $\bar{B}_s^0$ can mix through weak interaction, i.e. B_s^0-$\bar{B}_s^0$ oscillation. By experimental observation we can know whether CP is conserved. For the B_s^0-$\bar{B}_s^0$ system, the CP asymmetry is time dependent [3, 13]:

$$A_{CP}(t) \simeq A_{CP}^{dir}\cos(\Delta m t) + a_{\epsilon+\epsilon'}\sin(\Delta m t), \tag{16}$$

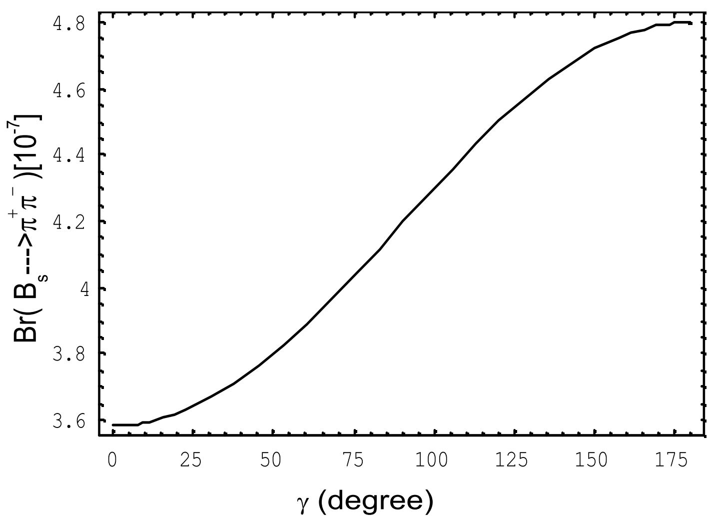

FIG. 2: The CP-averaged branching ratio of $B_s^0(\bar{B}_s) \to \pi^+\pi^-$ decay as a function of CKM angle γ.

where Δm is the mass difference of the two mass eigenstates of B_s mesons. A_{CP}^{dir} is the direct CP violation parameter while $a_{\epsilon+\epsilon'}$ is the mixing-related CP violation parameter. The direct CP violation parameter is defined as

$$A_{CP}^{dir} = \frac{\Gamma(B_s^0 \to \pi^+\pi^-) - \Gamma(\bar{B}_s^0 \to \pi^+\pi^-)}{\Gamma(B_s^0 \to \pi^+\pi^-) + \Gamma(\bar{B}_s^0 \to \pi^+\pi^-)} = \frac{2z\sin\gamma\sin\delta}{1+2z\cos\gamma\cos\delta + z^2}. \tag{17}$$

Using Eq.(9) and (10), we can compute the parameter A_{CP}^{dir}. The direct CP asymmetry A_{CP}^{dir} has a strong dependence on the CKM angle, as can be seen easily from Eq.(17) and Fig. 3. From this figure one can see that when the CKM angle γ is around $\pi/2$ the direct CP asymmetry reaches its peak, which is about 3%. The small direct CP asymmetry is also a result of small tree level contribution.

The mixing-related CP violation parameter in Eq.(16) is defined as [9]

$$a_{\epsilon+\epsilon'} = \frac{-2Im(\lambda_{CP})}{1+|\lambda_{CP}|^2}, \tag{18}$$

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{ts} \langle f|H_{eff}|\bar{B}_s^0\rangle}{V_{tb}V_{ts}^* \langle f|H_{eff}|B_s^0\rangle}. \tag{19}$$

In Fig. 4, we study the mixing CP violation parameter $a_{\epsilon+\epsilon'}$ of the decay $B_s \to \pi^+\pi^-$ as a function of CKM angle γ, just like the case of direct CP violation, it is almost symmetric

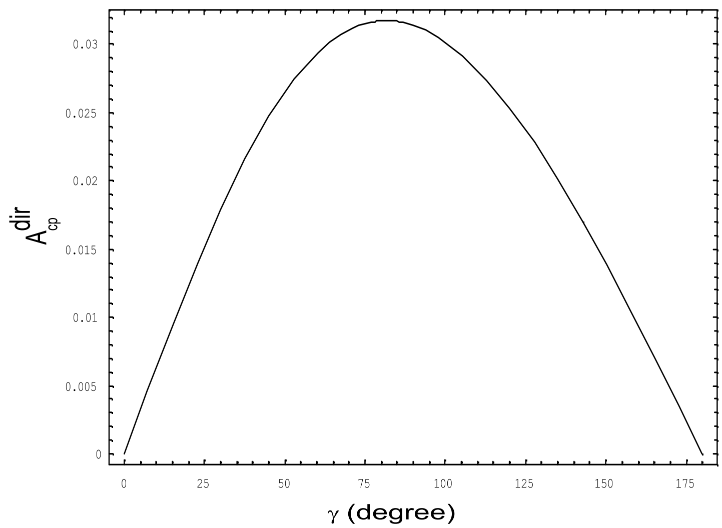

FIG. 3: Direct CP violation parameters of $B_s \to \pi^+\pi^-$ decay as a function of CKM angle γ.

and the symmetry axis is near $\gamma = \pi/2$. its peak is close to -14.5%. The possible large CP asymmetry might be observed at LHCb experiment in the future, this would help us to determine the value of CKM angle γ.

In conclusion, we study the branching ratio and CP asymmetry of the decay $B_s^0(\bar{B}_s^0) \to \pi^+\pi^-$ in PQCD, we find that the branching ratio is at the order of 10^{-7} and there are large CP asymmetries in the process, which may be measured in the future LHC-b experiments and BTeV experiment at Fermilab. This small branching ratio, predicted in the SM, make it sensitive to possible new physics contribution.

Acknowledgments

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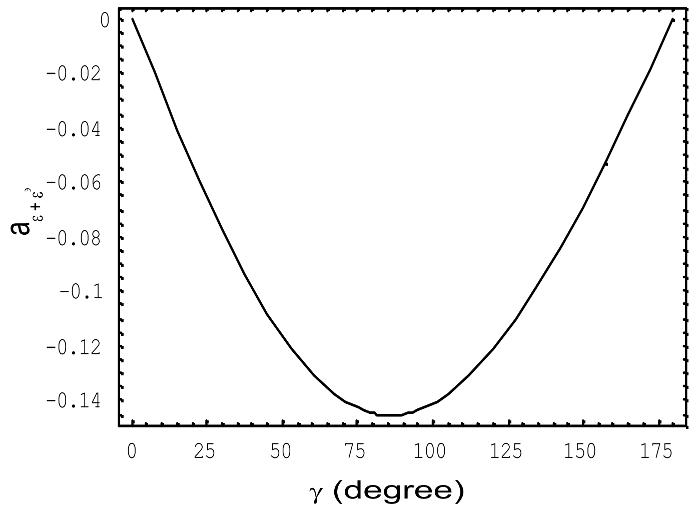

FIG. 4: Mixing-related CP violation parameter $a_{\epsilon+\epsilon'}$ of $B_s^0(\bar{B}_s) \to \pi^+\pi^-$ decay as a function of CKM angle γ.

APPENDIX A: SOME FORMULAS USED IN THE TEXT

For B_s meson wave function, we use the same wave function as B meson [9, 14], despite the possible SU(3) breaking effect

$$\phi_{B_s}(x,b) = N_{B_s} x^2(1-x)^2 \exp\left[-\frac{M_{B_s}^2\ x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right]. \tag{A1}$$

The parameter $\omega_b = 0.4$ GeV, is constrained by other charmless B decays [9, 14]. And $N_{B_s} = 114.0$GeV is the normalization constant using $f_{B_s} = 236$MeV.

The π meson's distribution amplitudes are given by light cone QCD sum rules [15]:

$$\begin{aligned}
\phi_\pi^A(x) &= \frac{3f_\pi}{\sqrt{2N_c}} x(1-x)\left\{1 + 0.44C_2^{3/2}(t) + 0.25C_4^{3/2}(t)\right\}, \\
\phi_\pi^P(x) &= \frac{f_\pi}{2\sqrt{2N_c}}\left\{1 + 0.43C_2^{1/2}(t) + 0.09C_4^{1/2}(t)\right\}, \\
\phi_\pi^T(x) &= \frac{f_\pi}{2\sqrt{2N_c}}(1-2x)\left\{1 + 0.55(10x^2 - 10x + 1)\right\},
\end{aligned} \tag{A2}$$

where $t = 1 - 2x$. The Gegenbauer polynomials are defined by:

$$\begin{aligned}
C_2^{1/2}(t) &= \frac{1}{2}(3t^2 - 1), \qquad C_4^{1/2}(t) = \frac{1}{8}(35t^4 - 30t^2 + 3), \\
C_2^{3/2}(t) &= \frac{3}{2}(5t^2 - 1), \qquad C_4^{3/2}(t) = \frac{15}{8}(21t^4 - 14t^2 + 1).
\end{aligned} \tag{A3}$$

Since the hard part is calculated only to leading order of α_s, we use the one loop expression for the strong running coupling constant in our numerical analysis,

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}, \tag{A4}$$

where $\beta_0 = (33 - 2n_f)/3$ and n_f is the number of active quark flavor at the appropriate scale μ. Λ is the QCD scale, we set $\Lambda = 250$MeV at $n_f = 4$.

The function $E_m(t_i)$ in Eq.(6) are defined by

$$E_m(t_i) = e^{-S_B(t_i) - S_{\pi^+}(t_i) - S_{\pi^-}(t_i)}, \tag{A5}$$

where S_B, S_{π^+}, S_{π^-} result from summing both double logarithms due to infrared gluon corrections and single ones caused by the renormalization of ultra-violet divergence. They are defined as:

$$S_B(t) = s(x_1 P_1^+, b_1) + 2\int_{1/b_1}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \tag{A6}$$

$$S_{\pi^-}(t) = s(x_2 P_2^+, b_2) + s((1-x_2)P_2^+, b_2) + 2\int_{1/b_2}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) \tag{A7}$$

$$S_{\pi^+}(t) = s(x_3 P_3^-, b_3) + s((1-x_3)P_3^-, b_3) + 2\int_{1/b_3}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \tag{A8}$$

where $s(Q,b)$ called Sudakov factor is given as [16]

$$s(Q,b) = \int_{1/b}^{Q} \frac{d\mu}{\mu} \left[\ln\left(\frac{Q}{\mu}\right) A(\alpha(\bar{\mu})) + B(\alpha_s(\bar{\mu})) \right] \tag{A9}$$

with

$$A = C_F \frac{\alpha_s}{\pi} + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{2}{3}\beta_0 \ln\left(\frac{e^{\gamma_E}}{2}\right) \right] \left(\frac{\alpha_s}{\pi}\right)^2, \tag{A10}$$

$$B = \frac{2}{3}\frac{\alpha_s}{\pi} \ln\left(\frac{e^{2\gamma_E - 1}}{2}\right) \tag{A11}$$

here γ_E is the Euler constant, n_f is the active flavor number.

The functions $h_a^{(1)}$, and $h_a^{(2)}$ in Eq.(6) come from the Fourier transformation of propagators of virtual quark and gluon. They are defined by

$$\begin{aligned} & h_a^{(j)}(x_1, x_2, x_3, b_1, b_2) = \\ & \left\{ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{x_2 x_3}\, b_1) \mathrm{J}_0(M_B\sqrt{x_2 x_3}\, b_2) \theta(b_1 - b_2) \right. \\ & \qquad \left. + (b_1 \leftrightarrow b_2) \right\} \times \begin{pmatrix} \mathrm{K}_0(M_B F_{(j)} b_1), & \text{for} \quad F_{(j)}^2 > 0 \\ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{|F_{(j)}^2|}\, b_1), & \text{for} \quad F_{(j)}^2 < 0 \end{pmatrix}, \end{aligned} \tag{A12}$$

where $\mathrm{H}_0^{(1)}(z) = \mathrm{J}_0(z) + i\,\mathrm{Y}_0(z)$, and $F_{(j)}$s are defined by

$$F^2_{(1)} = x_1x_3 - x_2x_3,\;\; F^2_{(2)} = x_1 + x_2 + x_3 - x_1x_3 - x_2x_3. \tag{A13}$$

The hard scale $t_i (i = 1, 2)$ in Eq.(6) are taken as the largest energy scale in the H to kill the large logarithmic radiative corrections:

$$t_i = \max(M_B\sqrt{|F^2_{(i)}|}, M_B\sqrt{x_2x_3}, 1/b_1, 1/b_2) \qquad (i = 1, 2). \tag{A14}$$

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